Electron–phonon relaxation in disordered two-dimensional electron gas with dynamically

screened deformation potential

Corrigendum

Electron–phonon relaxation in disordered two-dimensional electron gas with dynamically screened deformation potential

S S Z Ashraf, P Tripathi, A C Sharma and S T Hasan

2009 *J. Phys.: Condens. Matter* **21** 025504

The authors have noticed that equation (8) in the above article was displayed incorrectly which was an inadvertent typographical mistake on their part. The corrected version of this equation is as follows:

$$
\frac{1}{\tau_{e-ph}} = \left(\frac{D_0^2 k_B{}^3 T^3}{2\pi^2 \rho v_F \hbar^4 u^4} \right) \int_0^\infty dx\, x^2 \int_0^{\pi/2} d\theta
$$

$$
\times \left(\frac{xz \sin\theta (x^4 y^4 \sin^4\theta + 4x^2 y^2 \left(\frac{u}{v_F}\right)^2)}{xz \sin\theta (x^4 y^4 \sin^4\theta + 4x^2 y^2 \left(\frac{u}{v_F}\right)^2) + x^4 y^4 \sin^4\theta} \right)^2
$$

$$
\times \mathrm{Re} \left\{ \frac{xy \sin\theta}{\sqrt{(1 - ixy\left(\frac{u}{v_F}\right))^2 + x^2 y^2 \sin^2\theta - 1}} \right\} \{ N_x^{eq} + n_x^{eq} \}.
$$

$$(8)$$

However, the computations of the results were performed using the correct equation and hence the conclusions remain the same.

Electron–phonon relaxation in disordered two-dimensional electron gas with dynamically screened deformation potential

S S Z Ashraf[1]**, P Tripathi**[2]**, A C Sharma**[2] **and S T Hasan**[1]

[1] Physics Department, Shibli National P G College, Azamgarh 276001, India
[2] Physics Department, M S University of Baroda, Vadodara 390002, India

Abstract
We study the effect of a dynamically screened deformation potential on the electron longitudinal phonon relaxation in a disordered two-dimensional electron gas. On consideration of the dynamic dielectric function and polarization operator, and the frequency ω dependence, we find a significant change in the temperature exponent as well as the pre-factor α from the earlier reported approximate temperature power law dependence αT^4 obtained under static strong screening and impurity limit. More strikingly, a reversal in the character of the dependence of scattering rate on the mean free path takes place on the incorporation of dynamic screening, where the behaviour changes from the static $1/l$ to the dynamic l^2 at $T = 1.0$ K and $l = 10$ nm.

1. Introduction

The quantum transport properties of mesoscopic and disordered systems are governed by several quantum interference phenomena that involve electron dephasing processes. The dephasing time is the time interval until which the electron maintains its exact single-electron energy eigenstate in the presence of static impurities [1, 2]. The electronic state is changed because of transitions that occur between the energy eigenstates due to electron–electron (e–e), electron–phonon (e–ph) and impurities having internal degrees of freedom like dynamical defects or electron–magnetic–impurity interactions. These interactions influence the dephasing (phase breaking) and other relaxation processes and they significantly affect both the electronic transport as well as the temperature dependences of disordered mesoscopic systems [2, 3]. At low temperatures, when the elastic scattering rate dominates the e–e scattering, the electronic motion is diffusive between scattering events with other electrons. The diffusive motion of the electron causes it to remain in the interaction region for a longer time and thereby increases the interaction time. Because of this both the e–e and e–ph interactions are considerably enhanced in bulk and nanosystems [4].

The e–ph interaction problem in impure conductors has been considered in the literature for about three decades. Over

the last few years the problem has been revisited. But despite progress in the theory the issue has not been settled completely yet [2, 4, 5]. No systematic experimental investigation on the disorder-induced modification of relaxation through the e–ph interaction in semiconductors may possible be one of the reasons [6]. The e–ph interaction in disordered systems is much more complicated than in pure bulk systems. Even for the case of disordered systems the theory for metals does not work for semiconductors since the structure of the deformation potential (DP) differs for metals and semiconductors [7, 8]. For the case of acoustic scattering in semiconductors, DP and piezoelectricity are two important types of mechanism. The interaction between electrons and acoustic phonons via DP is most important above 2 K while below 2 K piezoelectric coupling is dominant, for a two-dimensional electron gas (2DEG) GaAs system [9, 10]. However, the temperature range for the dominant scattering mechanism depends upon the particular type of semiconductor. The DP interaction in semiconductors arises from local changes in the crystal's energy bands due to the lattice distortion caused by a phonon. The DP creates one of the major scattering mechanisms in non-polar semiconductors. Indeed, the DP interaction is a dominant source of electron energy loss in silicon-based electronic devices [11]. Nonetheless, the DP interaction is important for other semiconductor material devices as well.

1

Electron relaxation through the e–ph scattering via the DP in disordered semiconductor nanostructures has drawn considerable interest in recent years [6, 12, 13]. Sergeev *et al* have obtained analytical results on the e–ph scattering rates via the DP for nanostructures. Various temperature (T) and mean free path (l) dependences have been determined depending upon the amount of disorder and the contribution due to scattering of electrons by the static potential due to heavy defects and/or tough boundaries relative to the vibrating potential [12]. Nonetheless, it is known that, in the presence of a many-electron system, the Coulombic interaction is dynamically screened, which is a process whose behaviour is well captured by the random phase approximation dynamic dielectric function [14]. The significance of the dynamic screening has been emphasized in a number of papers [15–18]. Studies on the photoexcited populations have shown that the assumption of static screening underestimates the energy relaxation and therefore dynamic screening should be taken into account [15]. In the presence of strong disorder or high magnetic field, the reduced mean free path (and magnetic length) puts the 2D system into a regime where dynamic screening plays a crucial role [16]. Also, as the plasmon dispersion in 2DEG and 1DEG goes to zero as the momentum, q, gets smaller, dynamical effects are expected to be important for small q [19].

It therefore can be concluded from the above arguments that the dynamically screened DP in the presence of disorder is quite significant since it affects the electronic motion. Hence, we investigated the e–ph relaxation rate in a disordered 2DEG confined to a GaAs system interacting with bulk (3D) phonons in the diffusive limit incorporating the dynamic DP and polarization functions, to determine by what magnitude and to what degree and character the dynamical behaviour of these functions influence the relaxation rate. Our paper is organized as follows: section 2 describes the formalism, section 3 contains the results and discussion, and finally section 4 concludes the work.

2. Formalism

The e–ph relaxation rate, $\tau_{\mathrm{e-ph}}^{-1}$, is determined by the following kinetic equation [20, 21]:

$$\frac{1}{\tau_{\mathrm{e-ph}}} = -\frac{\delta}{\delta n(\varepsilon)}\frac{\mathrm{d}n(\varepsilon)}{\mathrm{d}t} \quad (1)$$

where $n(\varepsilon)$ is the non-equilibrium electron distribution function in a spatially uniform system. In equilibrium, the phonon and electron distribution functions, respectively, are $N_\omega = N_{\omega_q}^{\mathrm{eq}}$ and $n_\varepsilon = n_{\varepsilon_q}^{\mathrm{eq}}$, and $\tau_{\mathrm{e-ph}}^{-1}$ at the Fermi surface ($\varepsilon = 0$) is given by [12]

$$\frac{1}{\tau_{\mathrm{e-ph}}} = 4\tau \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^3} \frac{\gamma^2}{\left|\mathrm{Re}[\varepsilon(\omega_q, q)]\right|^2}$$
$$\times (N_{\omega_q}^{\mathrm{eq}} + n_{\omega_q}^{\mathrm{eq}}) \, \mathrm{Re}\left[\frac{\xi(\omega_q, q)}{1 - \xi(\omega_q, q)}\right] \quad (2)$$

in which τ is the electron momentum relaxation time due to electron impurity scattering, $\mathbf{q}$ is the 3D phonon momentum,

q is the characteristic momentum transferred to the electron due to the e–ph scattering, $\gamma = D_0(\mathbf{q}\cdot\hat{\mathbf{e}})/(2\rho\omega_q)^{1/2}$ is the unscreened vertex of the e–ph DP interaction, D_0 is the constant of the DP, $\hat{\mathbf{e}}$ is the polarization vector, ρ is the density of the material, $\omega_q = |q|u$, ω_q is the linear dispersion and u is the longitudinal sound velocity. $\mathrm{Re}[\varepsilon(\omega_q, q)]$ is the real part of the dynamic dielectric function and $\mathrm{Re}[\xi(\omega_q, q)/(1 - \xi(\omega_q, q))]$ is the real part of the polarization function that has been obtained by simplifying the response function, in which $\xi(\omega_q, q)$ is an integral over the impurity averaged electron Green functions.

Equation (2) is applicable to the case of arbitrary dimensionality. For determining the 2D e–ph relaxation rate we substitute in this equation the 2D dielectric function and polarization function and work out the scattering rate. For the case of 2DEG the polarization function $\xi(\omega_q, q)$ is determined from the following equation;

$$\xi(\omega, q_{ll}) = \frac{1}{\pi\nu\tau}\int\frac{\mathrm{d}\mathbf{p}}{(2\pi)^2} G^A(\mathbf{p}, \varepsilon) G^R(\mathbf{p}+q_{ll}, \varepsilon+\omega) \quad (3)$$

where q_{ll} is the phonon wavevector along 2DEG and ν is the electronic density of states. On integration and replacing ω by ω_q, equation (3) yields [22]

$$\xi(\omega_q, q_{ll}) = \frac{1}{\sqrt{(1 - \mathrm{i}\omega_q\tau)^2 + (q_{ll}l)^2}} \quad (4)$$

where $l = \nu_{\mathrm{F}}\tau$ is the electron mean free path due to scattering from impurities and ν_{F} is the Fermi velocity. For $q_{ll}l > u/\nu_{\mathrm{F}} \sim 10^{-2}$, ω dependence in ξ has been neglected in [12]. The screened DP involves the dielectric function defined as follows:

$$\varepsilon(\omega_q, q_{ll}) = 1 + V^0(q_{ll})\Pi(\omega_q, q_{ll}) \quad (5)$$

where $V^0(q_{ll}) = 2\pi e^2/\varepsilon_0 q_{ll}$ is the 2D Fourier transform of the bare Coulomb potential, e is the electronic charge, ε_0 is the background dielectric constant and $\Pi(\omega_q, q_{ll})$ is the polarization operator. In the limit of small q and ω, the dielectric function is given by [14]

$$\varepsilon(\omega_q, q_{ll}) = 1 + \frac{2\pi e^2}{\varepsilon_0 q_{ll}}\frac{\upsilon D q_{ll}^2}{-\mathrm{i}\omega_q + D q_{ll}^2} \quad (6)$$

where $D = \nu_{\mathrm{F}}^2\tau/2$ is the 2D diffusion constant. The 2D electronic density of states is given by $\nu = m^*/\pi\hbar^2$, where m^* is the effective electronic mass. For $\omega_q \ll D q_{ll}^2$, the ω_q dependence of the dielectric function can be dropped to yield

$$\varepsilon(q_{ll}) = 1 + \frac{k_2}{q_{ll}} \quad (7)$$

where $k_2 = 2\pi e^2\nu$ is the inverse screening length.

The relaxation rates have been calculated in [12] from equation (2) by substituting $\varepsilon(q_{ll})$ and $\xi(q_{ll})$, static dielectric and polarization functions. In terms of dimensionless parameters $x = \hbar qu/k_{\mathrm{B}}T$, $y = k_{\mathrm{B}}Tl/\hbar u$ and $z = k_{\mathrm{B}}T/\hbar u k_2$, where k_{B} and $\hbar$ are Boltzmann's and Planck's constants,

respectively, equation (2) with the use of equations (6) and (4) is given by

$$
\frac{1}{\tau_{e-ph}} = \left(\frac{D_0^2 k_B^3 T^3}{2\pi^2 \rho v_F \hbar^4 u^4} \right) \int_0^\infty dx\, x^2 \int_0^{\pi/2} d\theta
$$

$$
\times \left(\frac{xz\sin\theta(x^4 y^4 \sin^4\theta + 4x^2 y^2 (\frac{u}{v_F})^2) + x^4 y^4 \sin^4\theta}{xz\sin\theta(x^4 y^4 \sin^4\theta + 4x^2 y^2 (\frac{u}{v_F})^2)} \right)^2
$$

$$
\times \left(\mathrm{Re}\left\{ \frac{1}{\sqrt{(1 - i4xy(\frac{u}{v_F}))^2 + x^2 y^2 \sin^2\theta} - 1} \right\} \right)^2
$$

$$
\times \{N_x^{eq} + n_x^{eq}\}. \tag{8}
$$

Equation (8) is the elaborated equation for the electron relaxation rate incorporating the dynamic screened DP and dynamic polarization function in the diffusive limit (small q and ω).

3. Results and discussion

Approximate analytical results in the static weak and strong screening conditions and in quasi-ballistic ($q_{ll}l > 1$) and impure limits ($q_{ll}l < 1$) have earlier been reported for disordered nanostructures [12]. To trace the dependence of e–ph relaxation rates more rigorously on temperature and disorder, we took recourse to numerical calculations. Our computed results have been obtained for the cases of static as well as dynamic $\varepsilon(\omega_q, q_{ll})$ and $\xi(\omega_q, q_{ll})$ in the impure limit ($T < \hbar u/k_B l$). Our computed results are compared with earlier reported analytical results on τ_{e-ph}^{-1}:

$$
\tau_{e-ph}^{-1} = \pi^2 D_0^2 k_B^4 T^4 / (4\rho k_2^2 l v_F \hbar^5 u^5); \tag{9}
$$

which has been obtained under the approximation of strong screening and impure limit [12]. The τ_{e-ph}^{-1} has been computed as a function of T and l for disordered 2DEG confined to a GaAs system that has been modelled in terms of the following material parameters: $D_0 = 8.0$ eV, $\varepsilon_0 = 12.5$, $u = 5.2 \times 10^5$ cm s^{-1}, $\rho = 5.2$ g cm^{-3}, $m^* = 0.067m$ and electron density $n = 1 \times 10^{11}$ cm^{-2} [23]. The τ_{e-ph}^{-1} has been plotted in figures 1(a) and (b) as a function of T for two different values of $l = 3 \times 10^{-7}$ cm and $l = 3 \times 10^{-8}$ cm, respectively. Curve A in figure 1(a) displays the earlier reported analytical result for the static screening case (ω_q dependence is dropped in ε and ξ) and curve B represents our numerically computed results for the static screening case from equations (4) and (6) for $\varepsilon(q_{ll}, \omega_q = 0)$ and $\xi(q_{ll}, \omega_q = 0)$. Curve C has been obtained by computing equation (8) that includes the dynamic screening by taking $\varepsilon(q_{ll}, \omega_q)$ and $\xi(q_{ll}, \omega_q)$, where $\omega_q \neq 0$. From the inspection of figure 1(a), which is plotted for the mean free path $l = 3$ nm, we find that the inclusion of dynamic screening brings down the scattering rate by about an order to nearly two orders of magnitude for the lower and upper values of temperature, respectively, as compared to the static analytical result and also the exponent of the temperature decreases from T^4 for all values of temperature. The power exponent for curve B varies as ($T^{3.8}$–$T^{3.3}$) for the lower and upper values of the plotted range of temperatures. But the more noticeable change is in the magnitude and power dependence of curve C for which
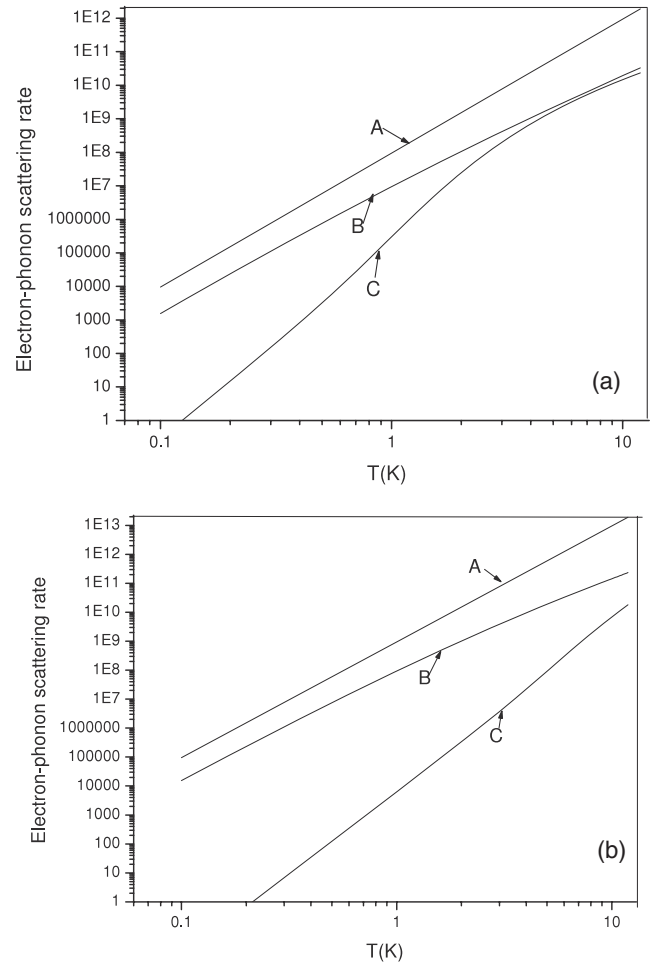

Figure 1. Electron–phonon scattering rate (τ_{e-ph}^{-1}) is plotted as a function of temperature (T) for mean free $l = 3$ nm (a) and $l = 0.3$ nm (b). Curve A displays the approximate analytical results reported in [12]. Curve B is for static ε and the ξ (ω_q dependence is dropped in equations (4) and (6)), while curve C is obtained using ε and ξ from equations (4) and (6).

the rate drops by three orders of magnitude for $T = 0.2$ K but afterwards the gap narrows and above 4 K curve C lies very much in close proximity to curve B. The merging of curve C with curve B for this value of $l(=3$ nm) and $T > 4$ K is due to the fact that the effect of the enhancement of scattering rates because of dynamic screening is approximately offset by the suppression of scattering rates due to dynamic $\xi(q_{ll}, \omega_q)$ and hence the curves lie very close to each other.

The enhancement in the scattering rate occurs due to plasmon excitation which is accounted for by the dynamic screening, but the damping is caused by impurity scattering that is incorporated in the polarization function. The impurity scattering gets further strengthened on the inclusion of frequency dependence in the polarization function. For $T > 4$ K and $l = 3$ nm, the overbearing of these two opposite effects on each other is not that conspicuous: however, for $T < 4$ K the suppression due to dynamic ξ is more than the enhancement by dynamic ε. The power dependence of curve C also fluctuates from T^6 to $T^{2.6}$ for the lower and upper limits of temperature. The DP is strongly screened
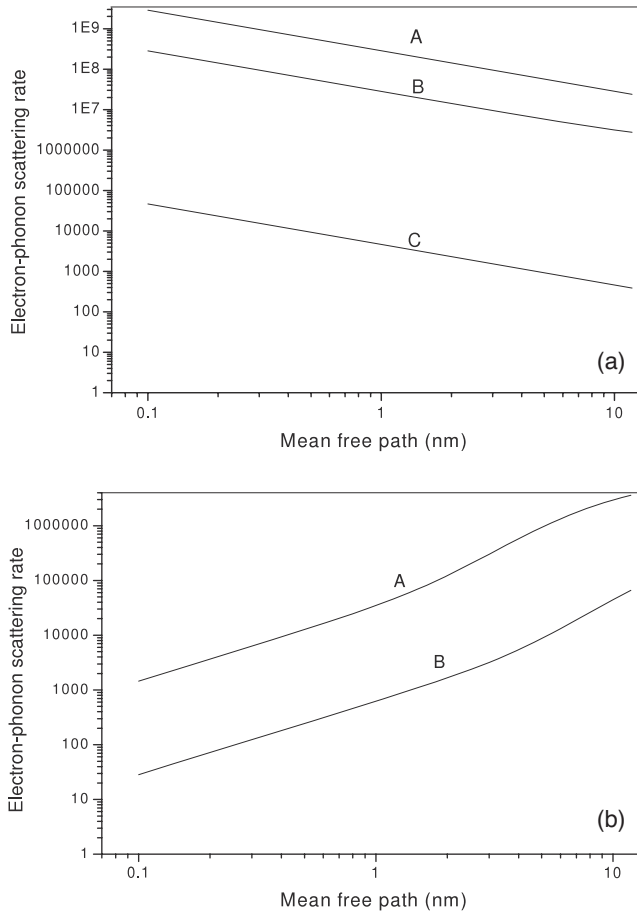
Figure 2. (a) Electron–phonon scattering rate ($\tau_{\text{e–ph}}^{-1}$) is plotted as a function of the mean free path, l. Curve A displays the analytical result reported in [12] at $T = 1$ K. Curve B and curve C are plotted by taking ω_q-independent ε and ξ at $T = 1$ K and $T = 0.1$ K, respectively. (b) Electron–phonon scattering rate ($\tau_{\text{e–ph}}^{-1}$) is plotted as a function of the mean free path, l, when dynamic screening is included in ε and ξ (ω_q dependence is included). Curve A is for $T = 1$ K while curve B is for $T = 0.5$ K.

at lower temperatures than at higher temperatures and hence the power exponent is larger at lower values of temperature but decreases from T^6 to $T^{2.6}$ as the temperature increases. However, figure 1(b), plotted for the case of relatively larger disorder ($l = 0.3$ nm), clearly demonstrates the pronounced effect of the dynamic ξ (around two orders of magnitude at 4 K) over the dynamic screening which strongly suggests the significant role that the dynamic screening and polarization play in disordered 2DEG systems. The temperature exponent of curve C escalates to $T^{5.98}$ from that of T^4 of the static case, curve A. Clearly the dynamic DP and polarization influences the results significantly.

The other important perspective of e–ph scattering (apart from the temperature dependence) is the dependence of scattering on the disorder or elastic mean free path l. Again, the dependence of relaxation on l in a disordered conductor is a contentious issue [2]. We plotted the scattering rate versus the mean free path both for static and dynamic cases, respectively, in figures 2(a) and (b). For the static case we find the exponent to vary inversely with l. But for the dynamic case

the scattering rate increases directly with the increasing amount of disorder which is also supported by experiments that have been conducted on GaAs/AlGaAs heterostructures [18, 24]. The dependence on the mean free path varies in the plotted limits of temperature as $l^{1.37}$–l^2 for curve A at $T = 1.0$ K and $l^{1.34}$–$l^{1.96}$ for curve B at $T = 0.5$ K. We therefore conclude from our theoretical study that there is a strong dependence of e–ph relaxation on dynamic DP and dynamic polarization function and we expect that the present work will provide quantitative information on the magnitude and character of $\tau_{\text{e–ph}}^{-1}(T, l)$ which may help experimental investigations.

4. Conclusions

The temperature and mean free path variations of $\tau_{\text{e–ph}}^{-1}$ depend strongly on disorder. The character of the electron–phonon interaction via the deformation potential (DP) is affected significantly by the inclusion of the dynamic screened DP and the dynamic polarization operator. On inclusion of the dynamic screened DP the temperature exponent is enhanced at lower temperatures and reduced at higher temperatures from that of the analytical T^4 result obtained under the statically screened DP approximation. The magnitude is also reduced by at most four orders of magnitude in the temperature range considered for $l = 0.3$ nm and the temperature dependence for this dynamic curve is $T^{5.98}$. More strikingly, a reversal in the character of the dependence of scattering on the mean free path takes place on the incorporation of dynamic screened DP where the behaviour changes from the static $1/l$ to dynamic l^2 at $T = 1.0$ K and $l = 10$ nm. Hence, it can be concluded from our study that the dynamic nature of screened DP plays a prominent role and governs strongly the temperature and mean free path dependence of the electron–phonon relaxation.

Acknowledgment

References

[1] Akkermans E and Montambaux G 2007 *Mesoscopic Physics of Electrons and Photons* (Cambridge: Cambridge University Press)
[2] Lin J J and Bird J P 2002 *J. Phys.: Condens. Matter* **14** R501–96
[3] Sergeev A, Yu Reizer M and Mitin V 2004 *Phys. Rev.* B **69** 075310
[4] Sergeev A, Karasik B S, Ptitsina N G, Chulkova G M, Il'in K S and Gershenzon E M 1999 *Physica* B **190** 263–4
[5] Hu S Y, Sheng P J and Lin J J 1999 *Phys. Rev.* B **60** 3940
[6] Yu X Z, Yang Y, Pan W and Shen W Z 2008 *Appl. Phys. Lett.* **92** 092106
[7] Sergeev A and Mitin V 2000 *Phys. Rev.* B **61** 6041
[8] Pipa V I, Vagidov N Z, Mitin V V and Stroscio M 2001 *Phys. Rev.* B **64** 235322
[9] Gantmakher V F and Levinson Y B 1987 *Carrier Scattering in Metals and Semiconductors* (Amsterdam: North-Holland)
[10] Ma Y, Fletcher R, Zaremba E, D'Iorio M, Foxon C T and Harris J J 1991 *Phys. Rev.* B **43** 9033

[11] Stroscio M A and Dutta M 2004 *Phonons in Nanostructures* (Cambridge: Cambridge University Press)

[12] Sergeev A, Yu Reizer M and Mitin V 2005 *Phys. Rev. Lett.* **94** 136602

[13] Prunnila M, Kivinen P, Savin A, Torma P and Ahopelto J 2005 *Phys. Rev. Lett.* **95** 206602

[14] Mahan G D 1990 *Many Particle Physics* 2nd edn (New York: Plenum)

[15] Chow E, Wei H P, Grivin S M and Shayegan M 1996 *Phys. Rev. Lett.* **77** 1143

[16] Lee S C and Galbraith I 1999 *Phys. Rev.* B **59** 15796

[17] Tripathi P and Ridley B K 2002 *Phys. Rev.* B **66** 195301

[18] Chow E, Wei H P, Grivin S M, Jan W and Cunningham J E 1997 *Phys. Rev.* B **56** R1676

[19] Ashraf S S Z and Sharma A C 2005 *J. Phys.: Condens. Matter* **17** 3043–59

[20] Yu Reizer M and Sergeev A V 1986 *Zh. Eksp. Teor. Fiz.* **90** 1056
Yu Reizer M and Sergeev A V 1986 *Sov. Phys.—JETP* **63** 616 (Engl. Transl.)

[21] Yu Reizer M 1989 *Phys. Rev.* B **40** 5411

[22] Sharma A C and Ashraf S S Z 2004 *J. Phys.: Condens. Matter* **16** 3117–32

[23] Glavin B A, Pipa V I, Mitin V V and Stroscio M A 2002 *Phys. Rev.* B **65** 205315

[24] Mittal A, Wheeler R G, Keller M W, Prober D E and Sacks R N 1996 *Surf. Sci.* **361/362** 537